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SUBJECT: A Discussion of the CINDA SCRPFPA
Subroutine and Related Theory
Case 620

DATE: September 30, 1969

FROM: G. M. Yanizeski
B. W. Lab

ABSTRACT

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SCRPFPA, a subroutine of the CINDA thermal computer program, calculates the "script F" values for a gray body enclosure divided into N isothermal surfaces. Script F is the fraction of radiant thermal energy emitted by a given surface that is absorbed by another surface including the interreflected energy components. The subroutine requires as input the areas, emissivities, and area-view factor products for the N surfaces. Calculations are based on the Oppenheim network representation.

(NASA-CR-106862) A DISCUSSION OF THE CINDA
SCRPFPA SUBROUTINE AND RELATED THEORY - CASE
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MEMORANDUM FOR FILE

INTRODUCTION

The CINDA* SCRPFPA subroutine, which calculates "script F", is important because of its direct applicability to many radiant heat transfer problems and particularly interesting because of its theoretical basis. Subroutine descriptions in the CINDA 3G manual are necessarily brief since there are over 300 subroutines listed. Therefore, SCRPFPA operation and usage are described in some detail in Section I and a sample problem is included in Section II. In addition, pertinent theoretical aspects, completely missing in the manual description, are presented in Section III.

I. GENERAL DESCRIPTION OF THE SUBROUTINE

SCRPFPA (CINDA - 3G Manual, p. 6.6.3) computes, for a gray body enclosure** consisting of N isothermal surfaces, the product $\sigma A_i F_{ij}$, where σ is any fixed number specified by the user (usually 1.0 or the Stefan-Boltzmann constant); A_i is the area of surface i, and F_{ij} , referred to as "script F," is the fraction of the radiant energy transfer from surface i that is adsorbed at surface j. (For a more rigorous definition of F_{ij} , see Eq. 3 under Theory.) In addition to σ , the surface areas A_i , emissivities ϵ_i and geometric view factors F_{ij} must be specified before using SCRPFPA.

Capacity

A maximum of "approximately" 300 surfaces can be considered on the UNIVAC 1108 (CINDA-3G Manual, p.6.6.2).

*CINDA - The "Chrysler Improved Numerical Difference Analyzer" computer program now in use by the Department 1022 Thermal Systems Group. The current "3G" version is written for third generation computers.

**By definition, a "gray" body absorbs a constant fraction of incident thermal radiation at all wave lengths. The limiting case is a "black" body which absorbs all of the incident thermal radiation. For additional discussion see Memorandum for File "Thermal Radiation Networks, with Examples of Lunar Ravine Heating" by D. P. Woodard.

Special Features

1. SCRPFA checks the size of input data arrays based on the number of surfaces N . If the arrays are too large or too small, the program prints a message and stops.
2. SCRPFA checks "allowable" emissivity values to avoid dividing by zero. If $\epsilon_1 < 0.0001$, then ϵ_1 is set equal to 0.0001, and the program continues. If $\epsilon_1 > 0.99999$, then ϵ_1 is set equal to 0.99999, and the program continues.
3. As in other CINDA subroutines, printed results are obtained only by calling another subroutine. The subroutine SYMLST used in the sample problem is recommended.

Calling Sequence - SCRPFA (AA(IC), A ϵ (IC), AFA(IC), ASFA(IC))

The terms AA(IC), A ϵ (IC), etc. appearing in the above calling sequence, are symbolic and actually are never used in the writing of problems. They have been employed in the CINDA-3G Manual to help the user associate the various arrays with their physical and mathematical counterparts. For an example of the actual notation employed, see the sample problem in this report.

Each of the terms in the calling sequence is defined as follows:

- IC - CINDA counts the number of elements in each array and these numbers, which are the "Integer Counts" (IC), then become the first elements in each array. (See "Array Data Block," p. 4.10, CINDA-3G Manual). This operation is automatically performed by CINDA and not by the user.
- AA - The areas A_i must be loaded into the Area Array (AA) before calling SCRPFA. For this array AA(1)=IC is given the value N by CINDA, and AA(I)= A_i where $I=2,3,\dots,N+1$ and $i=1,2,3,\dots,N$. In other words, $A_i=AA(i+1)$.
- A ϵ - The emissivities ϵ_i must be loaded into the Emissivity Array (A ϵ) as data before calling SCRPFA. For this array A ϵ (1) is given the value N by CINDA, and A ϵ (I)= ϵ_i

where again $I=2,3,\dots,N+1$ and $i=1,2,\dots,N$.

AFA- Again, CINDA sets $AFA(1)=IC$. The second value in AFA is σ . Next, the square matrix defined by the products $A_i F_{ij}$ is placed in AFA, a one-dimensional array. This storing of a square matrix in a one-dimensional array leads to added complexity in the SCRPFPA calculations, however, there is a substantial savings in storage that more than compensates. Since $A_i F_{ij} = A_j F_{ji}$, only the diagonal terms plus one set of off diagonal terms in the square matrix need to be stored. Thus AFA is termed the "Half FA" matrix array. When loading the $A_i F_{ij}$ values, the rows of the matrix are loaded in sequence beginning with the diagonal term $A_i F_{ii}$.

ASFA- The final results $\sigma A_i F_{ij}$ calculated by SCRPFPA are stored in ASFA. Again, since $A_i F_{ij} = A_j F_{ji}$, a half square matrix can be stored in a one-dimensional array. The results begin with $ASFA(3)$ equal to $\sigma A_i F_{ij}$. The manual (p.6.6.3) suggests calling subroutine SYMLST to "List the matrix values and identify them by row and column number."

II. SAMPLE PROBLEM

Card Description: (See Sample Data Deck)

Cards 1 - 22 - The data deck. Card 1 is blank.

Card 8 - Gives the first array A1 - hence the "1" as the first number listed. This corresponds to AA.

Cards 9 & 10 - Give the second and third arrays A2 and A3 corresponding to Ae and AFA - hence the "2" and "3". Note that other numbers for the arrays could have been chosen (i.e., A10, A12, etc.)

Card 13 - The subroutine SCRPFPA will be executed first. A1, A2 and A3 are the input arrays listed in cards 8, 9, and 10. The results will be returned in the A3 array which causes the original A3 array contents to be destroyed. (A3 is used as both AFA and ASFA.)

Card 14 - SYMLST will be called after SCRPFPA to list the results. A3+3 refers to the third element in the A3 array, the first actual calculated value. A3+1 refers to the first element, the IC count of the original A3 array.

SAMPLE DATA DECK

Col 8	Col 12	Card*
BCD	3GENERAL	\$ 2
BCD	9TEST SUBROUTINE SCRPEA	\$ 3
END		
BCD	3CONSTANTS DATA	\$ 5
END		\$ 6
BCD	3ARRAY DATA	\$ 7
	1,2,,3,,4,,3,,END	\$ 8
	2,,4,,2,,3,,5,,END	\$ 9
	3,4,1.0,,2,,0,,4,,6,,1,1.2,,9,1.2,1.2,,3,END	\$10
END		\$11
BCD	3EXECUTION	\$12
	SCRPEA(A1,A2,A3,A3)	\$13
	SYMLST(A3+3,A3+1)	\$14
END		\$15
BCD	3VARIABLES 1	\$16
END		\$17
BCD	3VARIABLES 2	\$18
END		\$19
BCD	3OUTPUT CALLS	\$20
END		\$21

*Symbols after dollar sign "\$" are ignored by CINDA

CHRYSLER IMPROVED NUMERICAL DIFFERENCING ANALYZER - C00045 (FORTRAN V VERSION)

TEST SUBROUTINE SCRPPFA

\$ 3

(1, 1)	1.41068-01	(1, 2)	1.32574-01	(1, 3)	2.06012-01	(1, 4)	3.19546-01	(
(2, 2)	0.01627-02	(2, 3)	1.77672-01	(2, 4)	2.29591-01	(
(3, 3)	3.40419-01	(3, 4)	4.75897-01	(
(4, 4)	4.74965-01	(

END OF DATA

@ FIN

ROUND: 0004 ACCOUNT: THR PROJECT: CINDA

TIME: 00:00:03.204 IN: 25 OUT: 0 PAGES: 15

INITIATION TIME: 12:40:13-SEP 26, 1969

TERMINATION TIME: 12:45:08-SEP 26, 1969

CORE-SECONDS: 260

IO COUNT: 637

CHARGE: 5.136

Other cards in the data deck are explained fully in the CINDA-3G Manual.

III. THEORY

Consider a gray body enclosure divided into N isothermal surfaces. The net heat flow Q_i from the i TH surface is*

$$Q_i = \frac{A_i \epsilon_i}{r_i} (E_i - J_i), \quad i = 1, 2, \dots, N \quad (1)$$

where

A_i = Area of surface i

ϵ_i = Emissivity of surface i

$r_i = 1 - \epsilon_i$ = Reflectivity of surface i ($\epsilon = \alpha$)

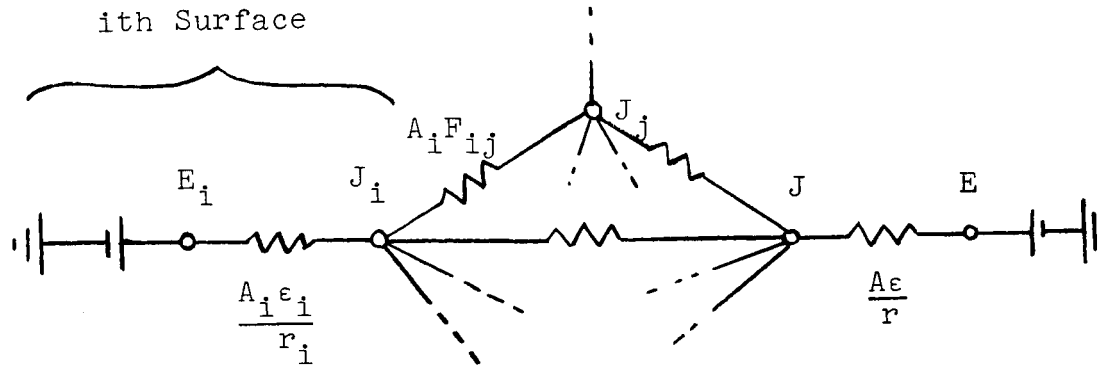
$E_i = \sigma T_i^4$ = Black body emissive power of surface i

J_i = "Radiosity" of Surface i = Sum of Reflected and Emitted Energy.

The enclosure of N surfaces and the modes of radiant heat transfer between them can be represented in a network of nodes and conductors.** As indicated by Eq. 1, in addition to node E_i an extra node, the radiosity node, J_i is needed for each surface. For example, consider the nodal representation of the ith surface:

* See, for example, Kreith, Radiation Heat Transfer, Scranton, Pa., Int. Textbook Co., 1962, pp. 46-47.

** Oppenheim, A. K., "Radiation Analysis by the Network Method," Trans. ASME, May 1956, p. 725.



where

F_{ij} = Geometric view factor or shape factor of Surface i with respect to Surface j .

The term $A_i F_{ij}$ can be considered the thermal conductance between the corresponding radiosity nodes. Similarly

$\frac{A_i \epsilon_i}{r_i}$ is the thermal conductance between E_i and J_i as indicated in Eq. 1.

It is desirable, although not necessary, to eliminate the radiosity nodes (J_i nodes) so that energy transfer between surfaces (between E nodes) can be calculated directly. Define F_{ij} so that

if

Q_{ij} = net heat flow from surface i to surface j

then

$$Q_{ij} = A_i F_{ij} [E_i - E_j]. \quad (3)$$

The term F_{ij} is "script F" which is distinctly different from the geometric view factor F_{ij} . Where F_{ij} is the fraction of energy emitted from i that is directly intercepted by j , F_{ij} is the fraction of energy emitted from i that is eventually absorbed by j including energy undergoing multiple reflections in the enclosure. Since the paths followed by emitted energy are fixed for a given gray body enclosure, F_{ij} is a constant independent of E_i and E_j .

If the F_{ij} values are determined for any convenient set of emissive powers, they are universally applicable. Therefore let

$$E_i = 1 \text{ (any surface } i\text{)}$$

and

$$E_j = 0, i \neq j \text{ (all other surfaces)}$$

Then all the energy entering any surface j must come from i , or

$$Q_j = Q_{ij}. \quad (4)$$

However from Eq. 1 rewritten for the energy entering surface j

$$Q_j = \frac{A_j \epsilon_j}{r_j} (J'_j - E_j) = \frac{A_j \epsilon_j}{r_j} J'_j \quad (5)$$

where the radiosity J'_j is intended as a particular value that only holds for the above choice of emissive powers. But by definition of F_{ij} :

$$\begin{aligned} Q_{ij} &= A_i F_{ij} (E_i - E_j) \\ &= A_i F_{ij} (1-0) \\ &= A_i F_{ij} \end{aligned} \quad (6)$$

Combining equations 4, 5 and 6 to solve for F_{ij} yields

$$F_{ij} = \frac{A_j}{A_i} \frac{\epsilon_j}{r_j} J'_j \quad (i \neq j) \quad (7)$$

As noted, Eq. 7 only holds for $i \neq j$; F_{ij} for $i = j$ will be discussed later.

Before employing Eq. 7, the various values for J_j' ($j=1, \dots, n$) must be calculated. The sum of the energy flows into any radiosity node J is zero, therefore writing an energy balance on each node J gives N simultaneous equations.* After some manipulation of these equations involving use of the reciprocity relationship ($A_i F_{ij} = A_j F_{ji}$) and the identity

$\sum_{j=1}^N F_{ij} = 1$, the following set of equations, presented here in matrix form, is obtained:**

$$\begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ \frac{A_1 \epsilon_1}{r_1} \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} = \begin{bmatrix} -A_1 F_{11} + \frac{A_1}{r_1} & -A_1 F_{12} & \dots & -A_1 F_{1n} \\ -A_2 F_{21} & -A_2 F_{22} + \frac{A_2}{r_2} & \dots & -A_2 F_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ -A_i F_{i1} & -A_i F_{i2} & \dots & -A_i F_{in} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ -A_n F_{n1} & -A_n F_{n2} & \dots & -A_n F_{nn} + \frac{A_n}{r_n} \end{bmatrix} \begin{bmatrix} J_1' \\ J_2' \\ \cdot \\ \cdot \\ J_i' \\ J_n' \end{bmatrix} \quad (8)$$

*This is an application of Kirchhoff's Law.

**A form of this equation was developed by Oppenheim, Op. cit. p. 728.

Note that the left column matrix has only one non-zero element corresponding to the only non-zero emissive power.

Letting capital letters represent matrices, Eq. (8) can be expressed more simply as:

$$A = SJ \quad (9)$$

Defining the inverse S^{-1} of S such that

$$S^{-1}S = SS^{-1} = I$$

Where I is the identity matrix. Eq. 9 can be solved for the radiosity matrix J with elements J'_j . Thus multiplying Eq. 9 by the inverse S^{-1} yields

$$S^{-1}A = IJ = J \quad (10)$$

Let s_{ij} be an element of S^{-1} . Then evaluating $J'_j = S^{-1}A$ yields the following expression for J'_j :

$$J'_j = \frac{A_i \epsilon_i}{r_i} s_{ji} \quad (11)$$

Substituting Equation (11) into Equation (7) results in the final expression for F_{ij} :

$$F_{ij} = A_j \frac{\epsilon_i \epsilon_j}{r_i r_j} s_{ji}, \quad i \neq j \quad (12)$$

(In CINDA, SCRPFA calls the subroutine SYMINV to invert S thus obtaining values for s_{ji} .) Note that $s_{ij} = s_{ji}$, therefore multiplying through Eq. (12) by A_i shows that $A_i F_{ij} = A_j F_{ji}$.

Eq. (12) is perfectly general since i and j are not specified, therefore, once S^{-1} has been determined, all $A_i F_{ij}$ for $i \neq j$ can be calculated simply by multiplying the various elements of S^{-1} by the appropriate values of $\frac{A_j \epsilon_i \epsilon_j}{r_i r_j}$.*

F_{ij} for $i = j$ must be calculated separately since Eq. (6) does not provide any information in this case. It can be shown, however, that F_{ij} for any i and j equals ϵ_i times the fractional part of the energy emitted by i that reaches j . In the special case where $i = j$, F_{ii} is ϵ_i times the fractional part of the energy leaving i that is returned to i (a mirror effect). Therefore

$$F_{ii} = \epsilon_i \frac{Q_{i \rightarrow i}}{Q_{\text{Total},i}} \quad (13)$$

However, by definition

$$Q_{i \rightarrow i} = Q_{\text{Total},i} - Q_i \quad (14)$$

where Q_i is the net energy leaving surface i as given by Eq. (1), and $Q_{\text{Total},i}$ is the total energy emitted by surface i , including $Q_{i \rightarrow i}$, which is returned. In addition, $Q_{\text{Total},i}$ is ϵ_i times the the black body emissive power of surface i , or

$$Q_{\text{Total}} = \epsilon_i E_i \quad (15)$$

Combining Eqs. (13), (14) and (15) and remembering that E_i has been assigned the value 1 yields the following expression for F_{ii} :

$$F_{ii} = \frac{\epsilon_i}{r_i} (J_i' - \epsilon_i) \quad (16)$$

*Ishimoto, T. and Bevans, J. T., "Method of Evaluating Script F for Radiant Exchange within an Enclosure," AIAA Journal, Vol. 1, June 1963, pp. 1428-29.

Substituting Eq. (11) for $i = j$ results in the value for F_{ii} calculated in SCRPFPA:

$$F_{ii} = \frac{\epsilon_i \epsilon_i}{r_i} \left(\frac{A_i}{r_i} s_{ii} - 1 \right) \quad (17)$$

It should be noted that in general F_{ii} is not zero as might be falsely concluded by examining Eq. (6).

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